CSCE476/876 **Spring 2005**

Homework 6

Assigned on: Monday, March 28, 2005.

Due: Monday, April 4, 2005.

This is a pen-and-paper homework, to be returned in class
Each question is worth 10 points. The whole homework is worth 40 points

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1 Scheduling problem as a CSP

Courtesy of Rina Dechter, UC-Irvine.

Consider the problem of scheduling five tasks: T1, T2, T3, T4 and T5, each of which takes one hour to complete. The tasks may start at 1:00, 2:00 or 3:00. Any number of tasks can be executed simultaneously provided the following restrictions are satisfied. T1 must start after T3. T3 must start before T4 and after T5. T2 cannot execute at the same time as T1 or T4. And T4 cannot start at 2:00.

1. Formulate the problem as a CSP by stating: the variables, their values and the constraints (hint: focus on the start time of a task.)
2. Draw the constraint graph.
3. Apply arc-consistency to each constraint in the CSP until no values can be ruled out (the CSP becomes arc-consistent).

2 Consistency checking

Consider the 4-Queens problem where each queen is associated with a row and can be assigned to any column in the row.

1. Express the domains of the variables.
2. Consider the binary constraints between every two queens and make explicit the constraint definition (i.e., give the constraint definition in extension).
3. Draw the constraint graph.

4. Arc-consistency of a binary constraint C_{Q_i,Q_j} between variables Q_i and Q_j ensures that every value for the variable Q_i has a support (at least one consistent value) in the domain of the variable Q_j, and vice versa. Run manually arc-consistency on the 4-Queens problem. Can you remove any value? At the end of the operation, the CSP is said to be arc-consistent.

5. Arc-consistency is also called 2-consistency because it considers all combinations of two variables at the same time. Let's consider *all combinations* of 3 variables at the same time. A CSP is said to be 3-consistent, if for every combination of consistent values for any two variables, one can find a value in the domain of any third variable such that the constraints between the three variables are satisfied. Is the 4-queen problem 3-consistent?

6. What can you conclude about the effectiveness of consistency algorithms for the 4-Queens problem?

3 Consistency checking

Courtesy of Rina Dechter, UC-Irvine.

Consider the simple coloring problem shown in Figure 1.

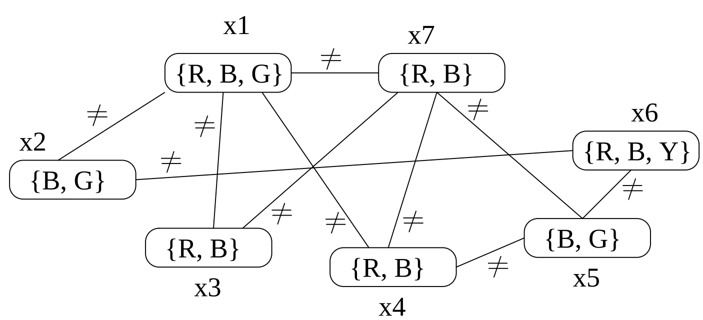

Figure 1: *A simple CSP.*

1. Compute the equivalent network that is *strong* 3-consistent
2. Find a solution to the CSP.

4 Latin square

Adapted from of Daphne Koller, Stanford University.

A Latin Square is a $N \times N$ array filled with colors, in which no color appears more than once in any row or column. Finding a solution to a 4×4 Latin Square can be formulated as a CSP, with a variable for each cell in the array, each having a domain of $\{r, g, b, y\}$, and a set of constraints asserting that any pair of cells appearing in the same row must have different colors, and that any pair of cells appearing in the same column must have different colors.

1 **r**	2 **g**	3 **b**	4 **y**
5 **g**	6 **y**	*r g b y*	r g b y
7 **b**	r g b y	r g b y	r g b y
r g b y	r g b y	r g b y	r g b y

Figure 2: *Current state of search.*

At this point in the search, seven of the cells have been instantiated (displayed in **boldface** in Figure 2), and the initial domains of the remaining cells are shown. The next cell to instantiate has the domain values in *italics*.

Re-execute (from scratch) the same 7 first assignments in the specified order and, at *each* assignment, draw the Latin Square while filtering the domains of the relative future variables. Do this process for the two following look-ahead strategies:

- the partial look-ahead strategy, forward checking (FC)
- the full look-ahead strategy, maintaining arc consistency.

Indicate eliminated values by crossing them out or just erasing them.