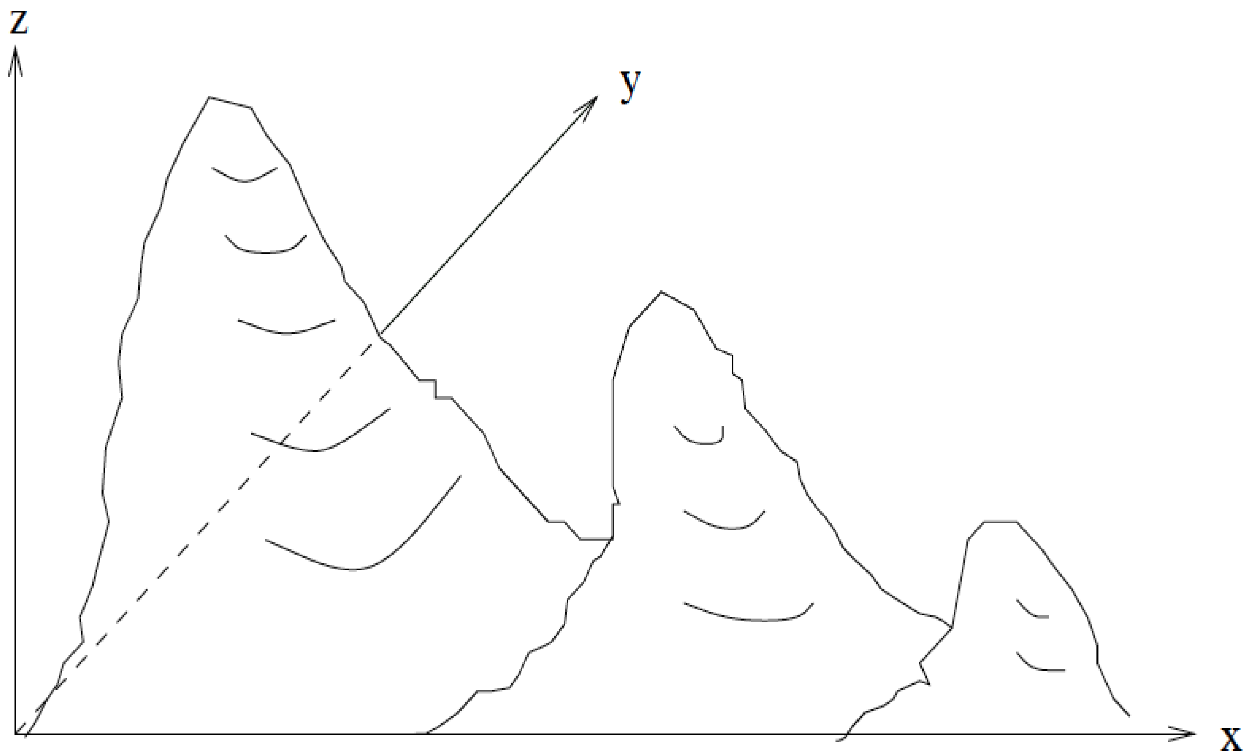
Interpolation and Approximation
(Chapter 18)

Peter Revesz

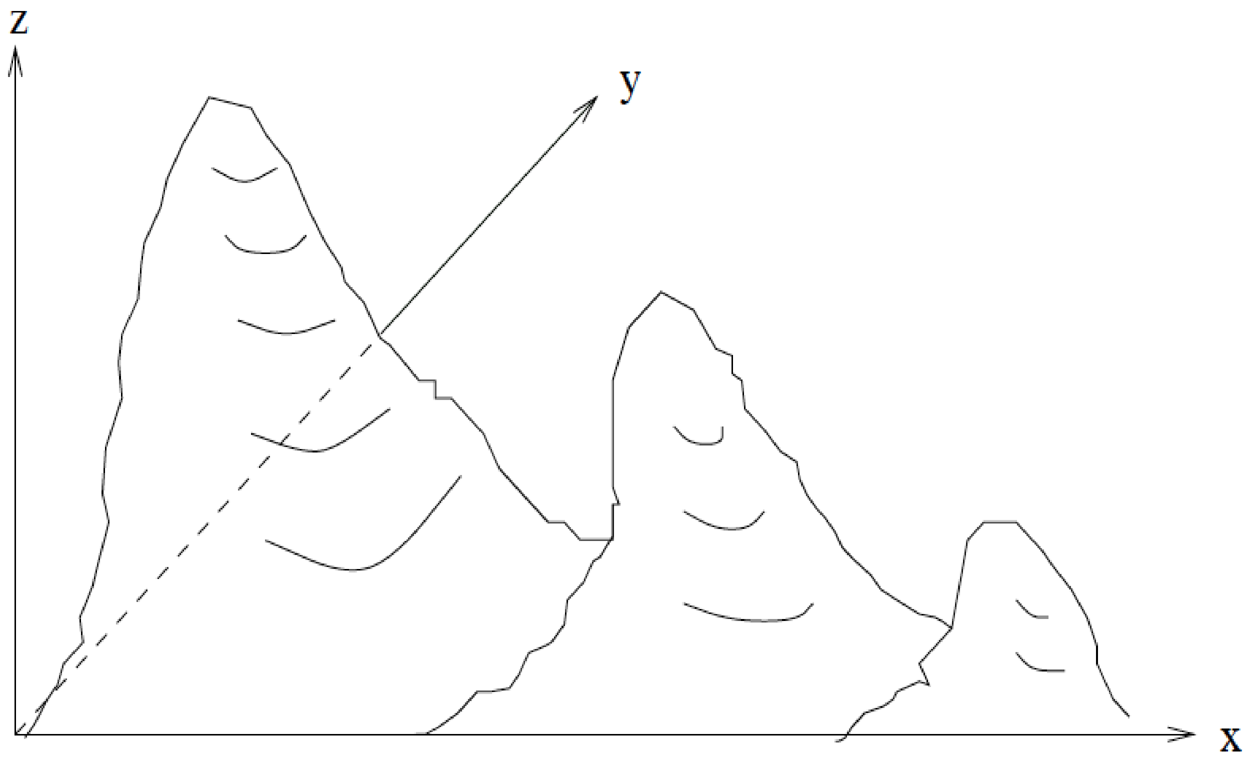
CSCE 413/813
Computer Science and Engineering
University of Nebraska – Lincoln

Triangulated Irregular Networks
(Section 18.2)



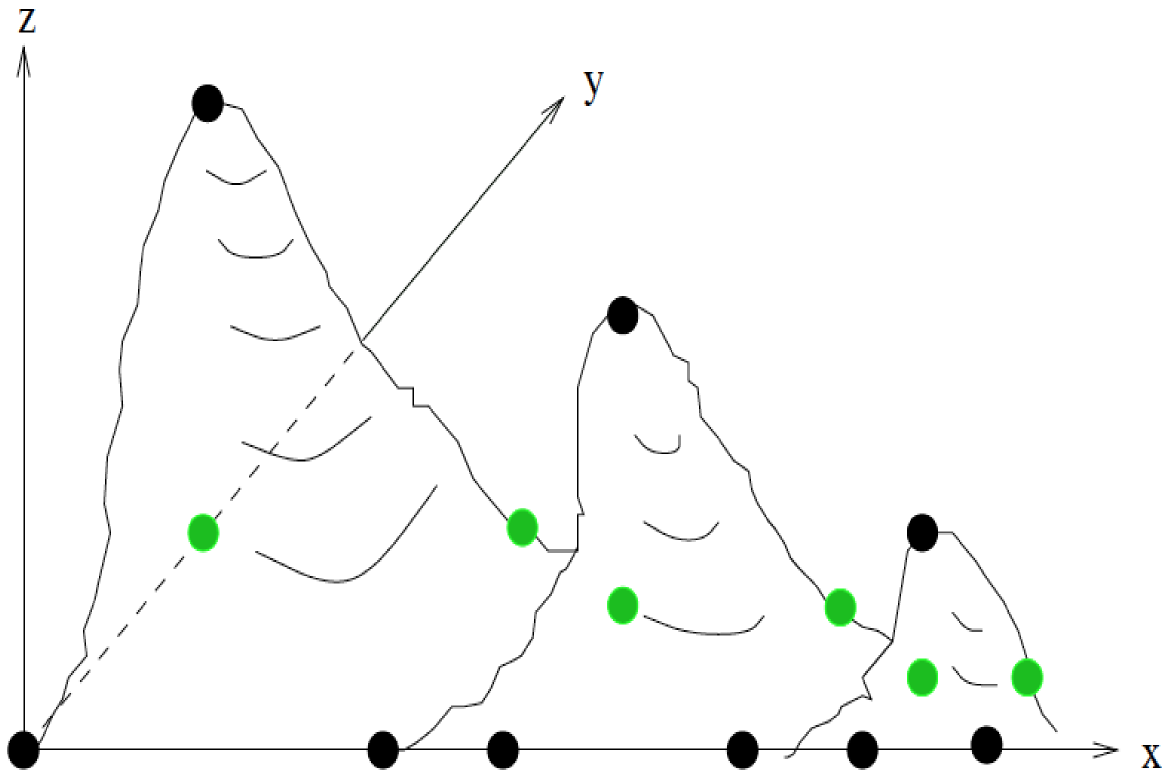
A mountain range.

Triangulated Irregular Networks
(Section 18.2)



A mountain range.

Triangulated Irregular Networks

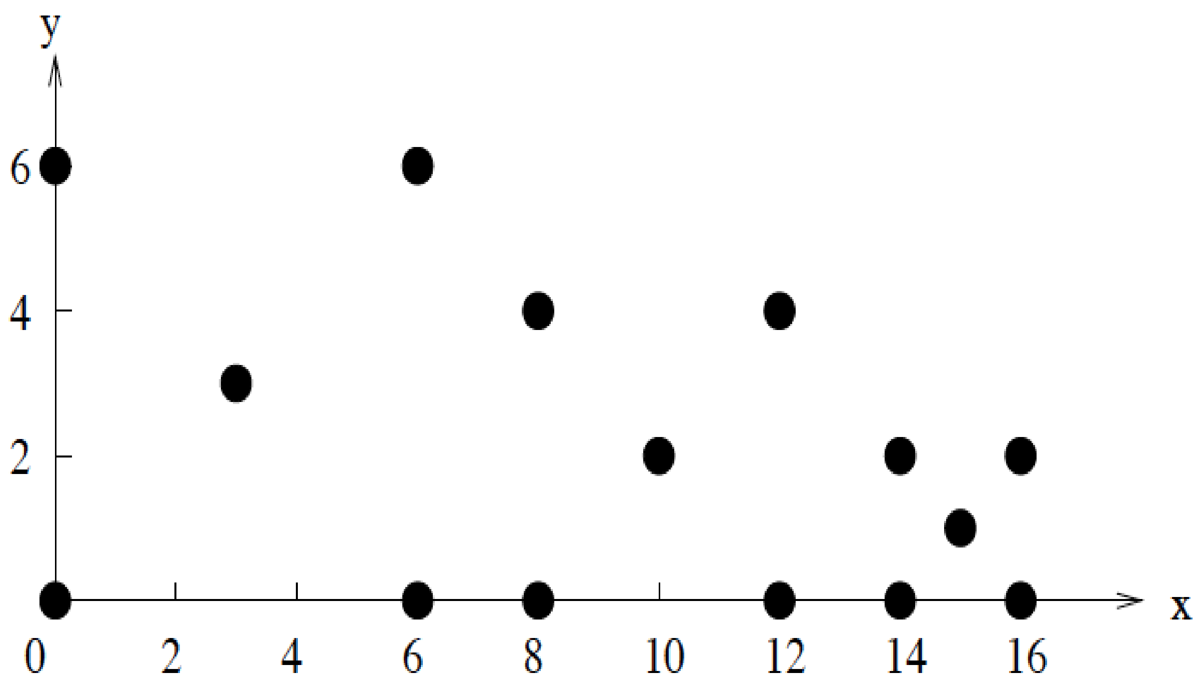


Sample_Points

ID	X	Y	Z
1	0	0	0
2	0	6	0
3	3	3	9
4	6	0	0
5	6	6	0
6	8	0	0
7	8	4	0
8	10	2	6
9	12	0	0
10	12	4	0
11	14	0	0
12	14	2	0
13	15	1	3
14	16	0	0
15	16	2	0

Sample points from the surface of the mountain range.

Triangulated Irregular Networks

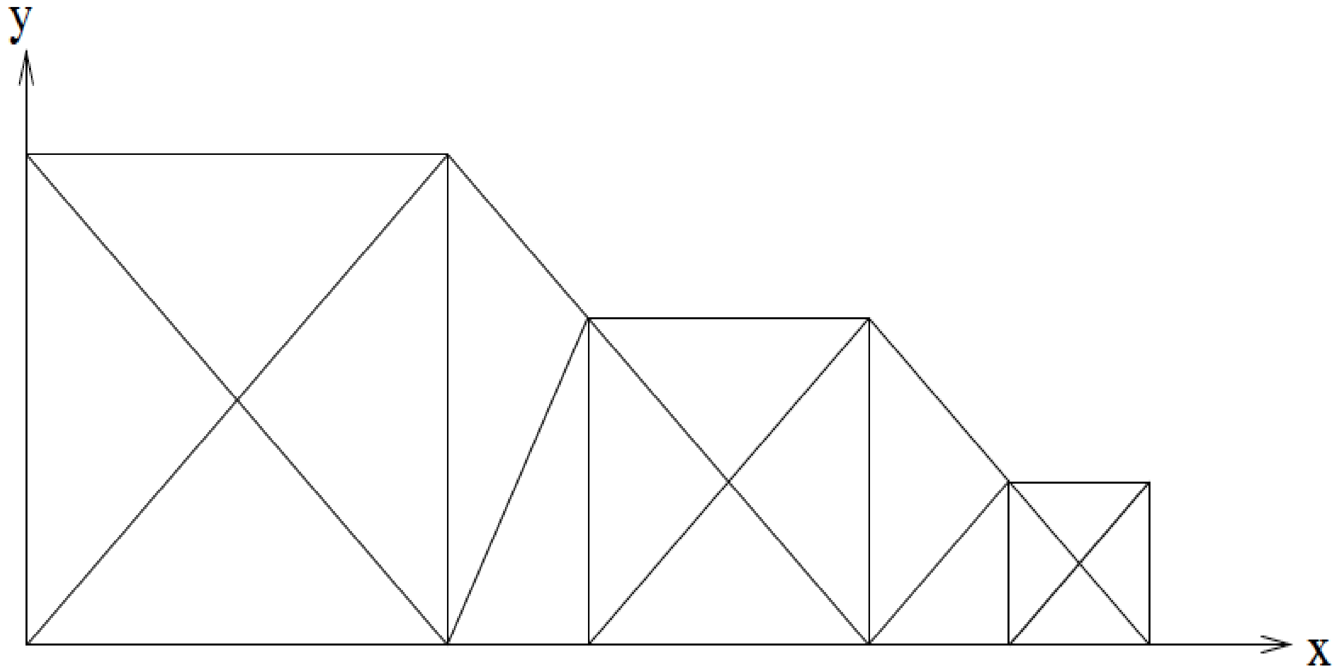


The (x,y) projection of the sample points.

Sample_Points

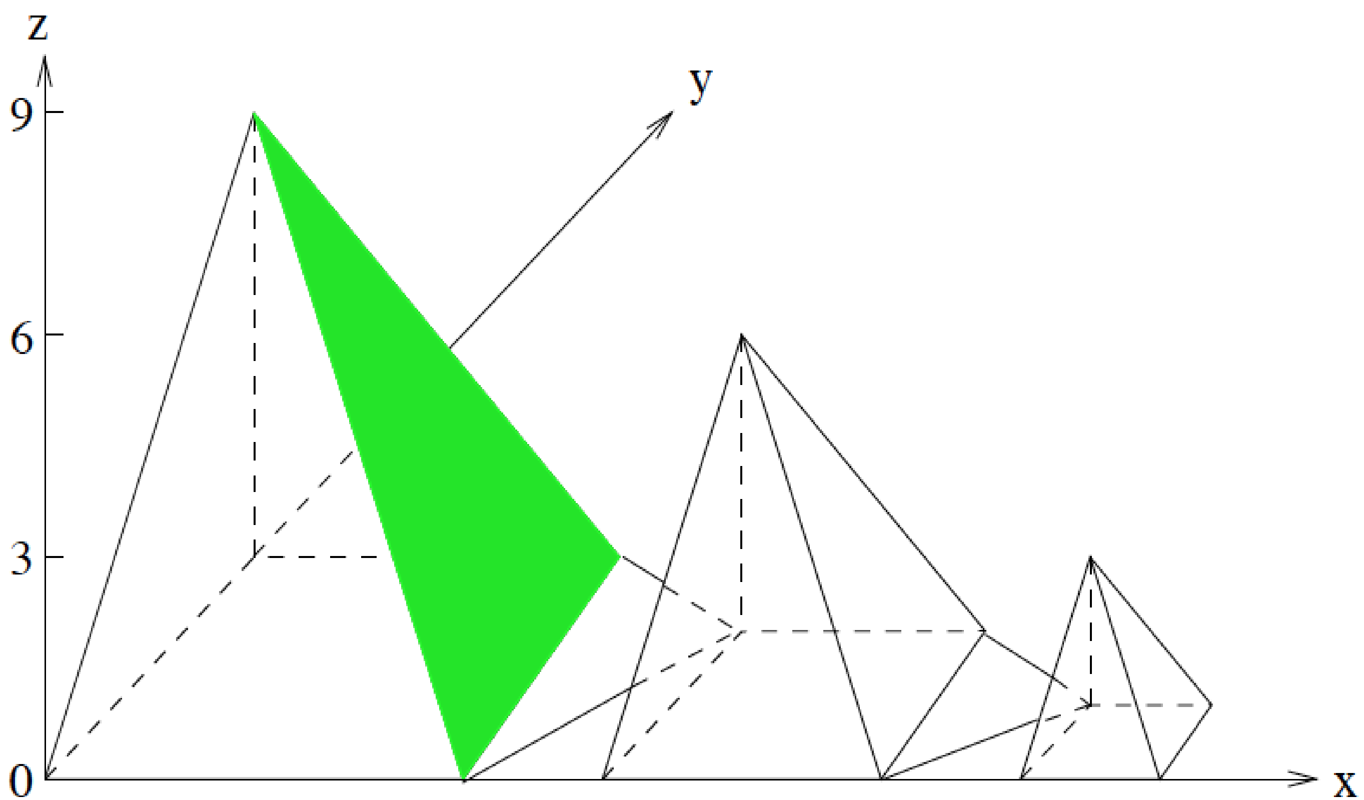
ID	X	Y	Z
1	0	0	0
2	0	6	0
3	3	3	9
4	6	0	0
5	6	6	0
6	8	0	0
7	8	4	0
8	10	2	6
9	12	0	0
10	12	4	0
11	14	0	0
12	14	2	0
13	15	1	3
14	16	0	0
15	16	2	0

Triangulated Irregular Networks



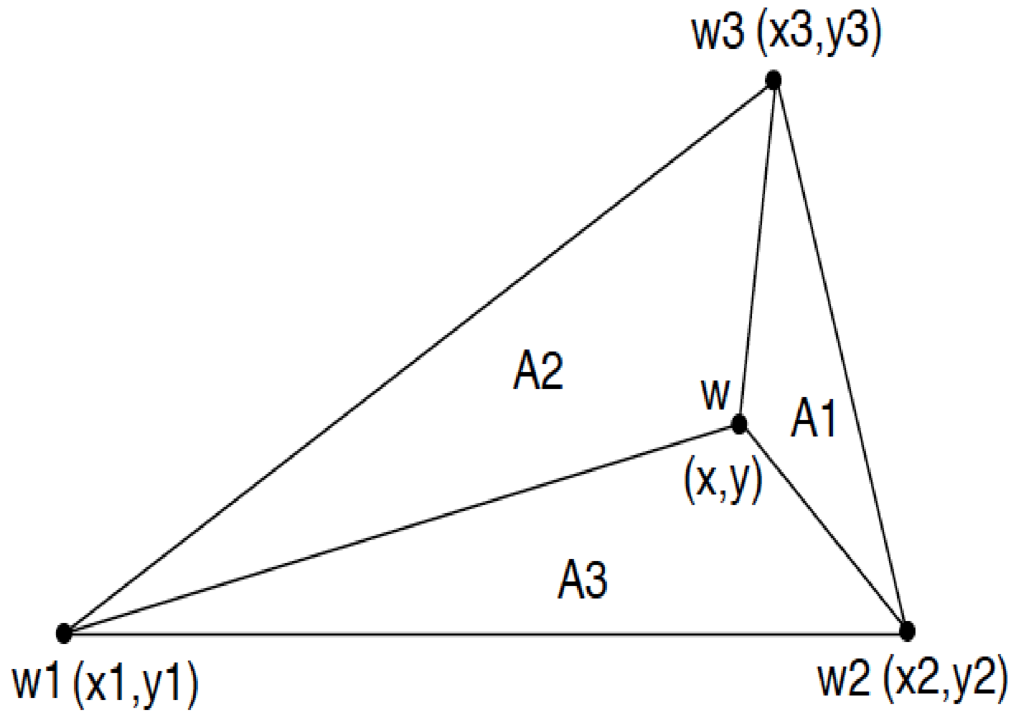
Triangulation of the sample points.

Triangulated Irregular Networks



The triangulated irregular network (TIN).

Shape Functions
(Section 18.3)



One element of the triangulated irregular network (TIN).

Shape Functions

$$w(x,y) = N_1(x,y)\, w_1 + N_2(x,y)\, w_2 + N_3(x,y)\, w_3$$

The coefficients are called shape functions because they are equivalent to ratios of the areas of the little triangles to the area of the whole triangle.

$$N_1(x,y) = \frac{x\,(y_2 - y_3) - y\,(x_2 - x_3) + (x_2\, y_3 - x_3\, y_2)}{2\mathcal{A}}$$

$$N_2(x,y) = \frac{x\,(y_3 - y_1) - y\,(x_3 - x_1) + (x_3\, y_1 - x_1\, y_3)}{2\mathcal{A}}$$

$$N_3(x,y) = \frac{x\,(y_1 - y_2) - y\,(x_1 - x_2) + (x_1\, y_2 - x_2\, y_1)}{2\mathcal{A}}$$

$$\mathcal{A} = \frac{1}{2}\, det \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$$

Shape Functions

Example: Let the measured values be 10, 20 and 30 at corner vertices (0,0), (10,0) and (10,5).

Find the shape function equation for the 3D triangle.

$$
\begin{aligned}
N_1(x,y) &= \frac{x(0-5) - y(10-10) + (10\times 5 - 10\times 0)}{2\times 25} = \frac{-x+10}{10} \\
N_2(x,y) &= \frac{x(5-0) - y(10-0) + (10\times 0 - 0\times 5)}{2\times 25} = \frac{x-2y}{10} \\
N_3(x,y) &= \frac{x(0-0) - y(0-10) + (0\times 0 - 10\times 0)}{2\times 25} = \frac{2y}{10}
\end{aligned}
$$

$$
w(x,y) = \frac{-x+10}{10}\times 10 + \frac{x-2y}{10}\times 20 + \frac{2y}{10}\times 30 = x + 2y + 10
$$

Shape Functions
(Example of Estimating Value at a Given Location)

Let the measured values be 18, 27 and 25 at corner vertices (2,1), (11,4) and (3,8). Find the value at location (6,4).

$$A = 30$$

$$w(6,4) = \frac{10}{30} \times 18 + \frac{12.5}{30} \times 27 + \frac{7.5}{30} \times 25 = 23.5$$

Alternatively, we can calculate the interpolation function and then substitute x = 6 and y = 4.

$$N_1(x,y)w_1 = \frac{-x-2y+19}{15} \times 18 = -1.2x - 2.4y + 22.8$$

$$N_2(x,y)w_2 = \frac{7x-y-13}{60} \times 27 = 3.15x - 0.45y - 5.85$$

$$N_3(x,y)w_3 = \frac{-x+3y-1}{20} \times 25 = -1.25x + 3.75y - 1.25$$

$$w(x,y) = 0.7x + 0.9y + 15.7$$