Piecewise Linear Approximation
(Section 18.6)

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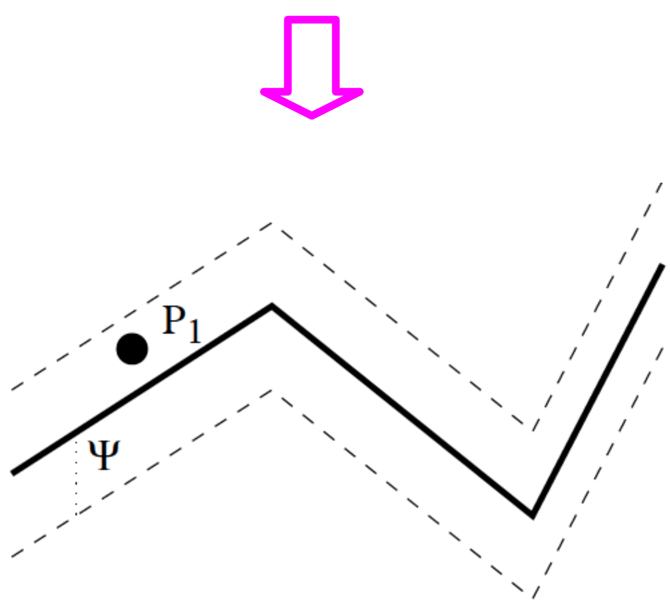
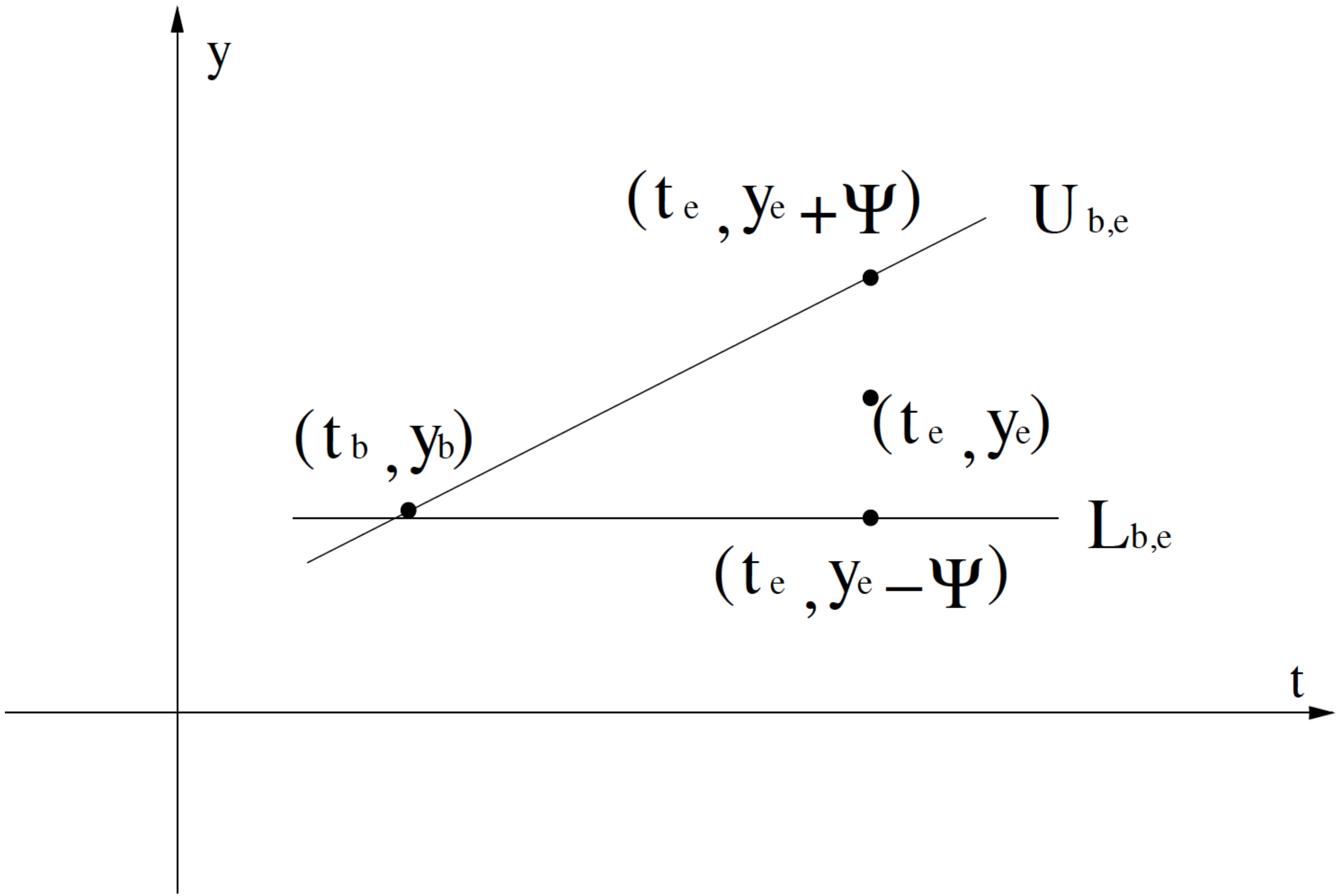
Approximation

The approximating function f(x) does not necessarily cross the original data points S. However, f(x) is within an error tolerance Ψ from the given data points.

$$|f(t_i) - y_i| \leq \Psi \text{ for each } (t_i, y_i) \in S$$



Piecewise Linear Approximation



Piecewise Linear Approximation(S,Ψ)

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input:      A time series S and a maximum error threshold Ψ.
output:     A piecewise linear approximation function.
local vars  Begin is start, S_L min and S_U max slope of a piece.
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$Begin := (t_1, y_1)$
$S_L := -\infty$
$S_U := +\infty$

for $i = 1$ to $(n-1)$ **do**
  $S'_L := \max(S_L,\ slope(Begin, (t_{i+1}, y_{i+1} - \Psi))$
  $S'_U := \min(S_U,\ slope(Begin, (t_{i+1}, y_{i+1} + \Psi))$
  if $S'_L \leq S'_U$ **then**
    $S_L := S'_L$
    $S_U := S'_U$
  else
    Add line $f(t) = \frac{S_L+S_U}{2}(t - Begin.t) + Begin.y$
    $Begin := (t_i, f(t_i))$
    $S_L := slope(Begin, (t_{i+1}, y_{i+1} - \Psi))$
    $S_U := slope(Begin, (t_{i+1}, y_{i+1} + \Psi))$
  end-if
end-for

Add line $f(t) = \frac{S_L+S_U}{2}(t - Begin.t) + Begin.y$

Find a Piecewise Linear Approximation with Ψ =5 for the data: (1, 68), (5, 71), (8, 76), (12, 73), (15, 71), (19, 68), (22, 70), (26, 74), (29, 73)

The algorithm works as follows. The initialization sets $Begin := (1, 68)$, $S_L := -\infty$ and $S_U := +\infty$. Next we see what happens in the *for loop* for successive values of i.

$i = 1$:

$$S'_L := \max(-\infty, \frac{(71-5)-68}{5-1}) = -0.5$$

$$S'_U := \min(+\infty, \frac{(71+5)-68}{5-1}) = 2$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

$i = 2$:

$$S'_L := \max(-0.5, \frac{(76-5)-68}{8-1}) = \max(-0.5, 3/7) = 3/7$$

$$S'_U := \min(2, \frac{(76+5)-68}{8-1}) = \min(2, 13/7) = 13/7$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

Find a Piecewise Linear Approximation with Ψ =5 for the data: (1, 68), (5, 71), (8, 76), (12, 73), (15, 71), (19, 68), (22, 70), (26, 74), (29, 73)

$i = 3$:

$$S'_L := \max(3/7, \frac{(73-5)-68}{12-1}) = \max(3/7, 0) = 3/7$$

$$S'_U := \min(13/7, \frac{(73+5)-68}{12-1}) = \min(13/7, 10/11) = 10/11$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

$i = 4$:

$$S'_L := \max(3/7, \frac{(71-5)-68}{15-1}) = \max(3/7, -1/7) = 3/7$$

$$S'_U := \min(10/11, \frac{(71+5)-68}{15-1}) = \min(10/11, 4/7) = 4/7$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

$i = 5$:

$$S'_L := \max(3/7, \frac{(68-5)-68}{19-1}) = \max(3/7, -5/18) = 3/7$$

$$S'_U := \min(4/7, \frac{(68+5)-68}{19-1}) = \min(4/7, 5/18) = 5/18$$

Because $3/7 > 5/18$, the *if* condition is false and the following line will be added as the first piece:

$$f(t) = 0.5(t-1) + 68$$

Find a Piecewise Linear Approximation with Ψ =5 for the data:
(1, 68), (5, 71), (8, 76), (12, 73), (15, 71), (19, 68), (22, 70), (26, 74), (29, 73)

We also set $Begin := (15, 75)$ and

$$S_L := \frac{(68-5)-75}{19-15} = -3$$

$$S_U := \frac{(68+5)-75}{19-15} = -0.5$$

$i = 6$:

$$S'_L := \max(-3, \frac{(70-5)-75}{22-15}) = \max(-3, -10/7) = -10/7$$

$$S'_U := \min(-0.5, \frac{(70+5)-75}{22-15}) = \min(-0.5, 0) = -0.5$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

$i = 7$:

$$S'_L := \max(-10/7, \frac{(74-5)-75}{26-15}) = \max(-10/7, -6/11) = -6/11$$

$$S'_U := \min(-0.5, \frac{(74+5)-75}{26-15}) = \min(-0.5, 4/11) = -0.5$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

Find a Piecewise Linear Approximation with Ψ =5 for the data: (1, 68), (5, 71), (8, 76), (12, 73), (15, 71), (19, 68), (22, 70), (26, 74), (29, 73)

$i = 8$:

$$S'_L := \max(-6/11, \frac{(73-5)-75}{29-15}) = \max(-6/11, -7/14) = -0.5$$

$$S'_U := \min(-0.5, \frac{(73+5)-75}{29-15}) = \min(-0.5, 3/14) = -0.5$$

The *if* condition is true, hence $S_L := S'_L$ and $S_U := S'_U$.

Now we exit the *for loop* and in the last line we create the piece:

$$f(t) = -0.5(t-15) + 75$$

Temperature

SN	T	Temp
1	1	68
1	5	71
1	8	76
1	12	73
1	15	71
1	19	68
1	22	70
1	26	74
1	29	73
2	2	71
2	4	69
2	9	68
2	11	66
2	16	70
2	18	68
2	23	71
2	25	75
2	30	77
3	6	75
3	13	78
3	20	72
3	27	68
4	7	72
4	14	75
4	21	72
4	28	74

Temperature2

SN	t	Temp	
1	t	y	$y = 0.5(t-1) + 68, \quad t \geq 1, \quad t \leq 15$
1	t	y	$y = -0.5(t-15) + 75, \quad t \geq 15, \quad t \leq 29$
2	t	y	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

PRACTICE: Find a Piecewise Linear Approximation with $\Psi = 5$ for the data:
(2, 71), (4, 69), (9, 68), (11, 66), (16, 70), (18, 68), (23, 71), (25, 75), (30, 77)

PRACTICE: Find a Piecewise Linear Approximation with Ψ =5 for the data: (2, 71), (4, 69), (9, 68), (11, 66), (16, 70), (18, 68), (23, 71), (25, 75), (30, 77)

i = 1

$$S_U = \min\left(+\infty, \frac{(69+5)-71}{4-2}\right) = 1.5 \qquad S_L = \max\left(-\infty, \frac{(69-5)-71}{4-2}\right) = -3.5$$

i = 2

$$S_U = \min\left(1.5, \frac{(68+5)-71}{9-2}\right) = \frac{2}{7} \qquad S_L = \max\left(-3.5, \frac{(68-5)-71}{9-2}\right) = -\frac{8}{7}$$

i = 3

$$S_U = \min\left(\frac{2}{7}, \frac{(66+5)-71}{11-2}\right) = 0 \qquad S_L = \max\left(\frac{-8}{7}, \frac{(66-5)-71}{11-2}\right) = -\frac{10}{9}$$

i = 4

$$S_U = \min\left(0, \frac{(70+5)-71}{16-2}\right) = 0 \qquad S_L = \max\left(\frac{-10}{9}, \frac{(70-5)-71}{16-2}\right) = -\frac{3}{7}$$

i = 5

$$S_U = \min\left(0, \frac{(68+5)-71}{18-2}\right) = 0 \qquad S_L = \max\left(\frac{-3}{7}, \frac{(68-5)-71}{18-2}\right) = -\frac{3}{7}$$

PRACTICE: Find a Piecewise Linear Approximation with Ψ =5 for the data: (2, 71), (4, 69), (9, 68), (11, 66), (16, 70), (18, 68), (23, 71), (25, 75), (30, 77)

i = 6

$$S_U = \min\left(0, \frac{(71+5)-71}{23-2}\right) = 0$$

$$S_L = \max\left(\frac{-3}{7}, \frac{(71-5)-71}{23-2}\right) = -\frac{5}{21}$$

i = 7

$$S_U = \min\left(0, \frac{(75+5)-71}{25-2}\right) = 0$$

$$S_L = \max\left(\frac{-5}{21}, \frac{(75-5)-71}{25-2}\right) = -\frac{1}{23}$$

i = 8

$$S_U = \min\left(0, \frac{(77+5)-71}{30-2}\right) = 0$$

$$S_L = \max\left(\frac{-1}{23}, \frac{(77-5)-71}{30-2}\right) = \frac{1}{28}$$

Since $S_U < S_L$ and $\text{avg}\left(0, \frac{-1}{23}\right) = \frac{-1}{46}$, the first equation is: $Y = -\frac{1}{46}(X-2) + 71$

This equation holds between points (2, 71) and (25, 70.5). Starting from the latter point, we continue to the last point (30, 77) with a slope of (77-70.5)/(30-25)=6.5/5=1.3

$$Y = 1.3(X - 25) + 70.5$$